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1991 J. Phys.: Condens. Matter 3 7085

(http://iopscience.iop.org/0953-8984/3/36/016)

J. Phys.: Condens. Matter 3 (1991) 7085–7089. Printed in the UK

LETTER TO THE EDITOR

Phase-driven current and quantum interference in the quantum Hall regime: II. More insight

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Received 25 March 1991

Abstract. Experimentally observed periodic oscillations in the magnetoresistance in the quantum Hall regime have been looked at in the framework of phase-driven current in narrow quantum-Hall samples. A modification to the previous calculation of the period of oscillations presented by this author is suggested and more insight into the phenomenon is given.

In a recent publication [1] (hereafter referred to as VS) we presented a plausible explanation for periodic oscillations in the magnetoresistance, R_{xx}, reported by Mottahedeh *et al* [2] for a narrow two-dimensional electron gas (2D EG) system under the quantum-Hall condition. The explanation was based on the possible existence of a phase-driven alternating current (AC) expected to be present in a system such as that which was suggested by this author previously [3, 4]. Here we present more insight into the mechanism proposed in VS for the oscillations in R_{xx}. This results from the re-examination of the expression (3) of VS, which was used to obtain the width of the region between the two edge currents in which the phase-driven AC flows. In particular we find that this 'width' can be controlled externally by adjusting the magnitude of the system current.

To recapitulate, recall that it was suggested that the AC, which emerges due to the coupling between the two edge currents, flows in the transverse direction (i.e., the y-direction) over a region, say δ-wide, spread in the middle of the 2D EG along the length, in the x-direction. If the edge currents are separated by a distance w, then δ will have to be a little larger than this. We had calculated the space- and time-dependent phase difference, $\varphi_{12}(x, t)$, between two points lying perpendicular to the x-axis, at the two edges of the δ-wide strip, which fall in the edge currents 1 and 2, respectively,

$$\varphi_{12}(x, t) = \varphi_{12}(0, 0) + (2\pi/\Phi_0)(Bx\delta + V_H t) \tag{1}$$

where B, Φ_0 and V_H denote the strength of the magnetic field, the flux quantum and the Hall voltage, respectively.

In VS we had taken δ to be the half-wave length of the AC assuming that the AC flows with the speed of light c (and the frequency of the AC $= eV_H/\hbar = 2\pi c/\lambda$, so that $\lambda/2 \equiv \delta = (hc/e)/2V_H = \Phi_0/2V_H$). We now realize, (i) that the AC may not flow with speed c, and (ii) that $\lambda/2 = \Phi_0/(2V_H)$ is incorrect, dimensionally, unless $c = 1$. Taking the speed of the AC to be v_y and that of the system current to be v_x (both v_x and v_y shall be calculated later), we make fresh calculations of δ and of the period, ΔB, of the oscillations in R_{xx}. We will proceed along the lines suggested in VS but in a more elaborate manner using figure 1 as an aid.

First, note that the direction of the phase-driven (transverse) current alternates periodically as one moves in the x-direction in accordance with the variation of φ_{12} in this direction. For instance, at a particular instant, say $t = 0$, φ_{12} varies with x as,

$$\varphi_{12}(x, 0) = \varphi_{12}(0, 0) + (2\pi/\Phi_0)B\delta x \tag{2}$$

so that the direction of the flow of the transverse current will flip at an interval Δx, given by,

$$\varphi_{12}(x, 0) - \varphi_{12}(0, 0) = n\pi = (2\pi/\Phi_0)B\delta n\Delta x \qquad n = 1, 2, 3, \ldots. \tag{3}$$

Furthermore, φ_{12} at a given point in space varies with time under the influence of V_{H} according to,

$$\varphi_{12}(x, t) = \varphi_{12}(x, 0) + (2\pi/\Phi_0)V_{\mathrm{H}}t \tag{4}$$

so the direction of flow at that point will flip at an interval Δt, given by,

$$\varphi_{12}(x, t) - \varphi_{12}(x, 0) = n\pi = (2\pi/\Phi_0)V_{\mathrm{H}}n\Delta t. \tag{5}$$

Now, suppose that an electron is at an arbitrarily chosen origin $x = 0$ at the time $t = 0$ (figure 1(a)), and also suppose that

$$\varphi_{12}(0, 0) \equiv \varphi_{12}^0 = \pi/2 \tag{6}$$

then the maximum phase-driven current, J_c will be flowing from side 1 to side 2 [1, 3], so the electron will be moving from side 2 to 1 with *maximum probability* [1]. This flow will continue until $t = \Phi_0/(4V_{\mathrm{H}})$ (the position coordinate remaining as $x = 0$) when J will be zero and the electron will lose all its y-momentum (figure 1(b)). At this stage the electron will move along with the edge current on side 1. In the time interval $(0, \Phi_0/4V_{\mathrm{H}})$ the distance travelled in the y-direction is

$$\delta = v_y\Phi_0/(4V_{\mathrm{H}}). \tag{7}$$

At $t = \Phi_0/(2V_{\mathrm{H}})$, the electron would have moved a distance Δx in the positive x-direction given by,

$$\varphi_{12}(\Delta x, (\Phi_0/2V_{\mathrm{H}})) = \varphi_{12}(0, 0) + (2\pi/\Phi_0)B\delta\Delta x + (2\pi/\Phi_0)V_{\mathrm{H}}(\Phi_0/2V_{\mathrm{H}}). \tag{8}$$

However, at this moment, electrons will be flowing from side 2 to 1 carrying the maximum current J_c (figure 1(c)), so the electron under consideration will move ahead in the positive x-direction. However, if v_x, the speed of the electrons carrying the system current, is such that at $t = \Phi_0/(2V_{\mathrm{H}})$ the distance travelled in the positive x-direction is $2\Delta x$, then the electron under consideration will cross over to side 2 with maximum probability (the transverse current at $x = 2\Delta x$ being $-J_c$: figure 1(c)). In this case, in accordance with figure 1(c), we shall have,

$$\varphi_{12}(2\Delta x, \Phi_0/2V_{\mathrm{H}}) = \pi/2 + (2\pi/\Phi_0)(B\delta 2\Delta x + V_{\mathrm{H}}(\Phi_0/2V_{\mathrm{H}})) = 7\pi/2. \tag{9}$$

i.e.

$$B\delta 2\Delta x = \Phi_0. \tag{10}$$

Thus the rectangular loop, $\delta(2\Delta x)$, performed by the electron encloses a flux quantum. Writing (10) in terms of v_x and v_y,

$$v_x v_y = 16V_{\mathrm{H}}^2/(B\Phi_0) = 50.36 \times 10^3 \tag{11}$$

for $V_{\mathrm{H}} = 13 \times 10^{-6}$ V corresponding to plateau $i = 2$ at $B \simeq 13$ T.

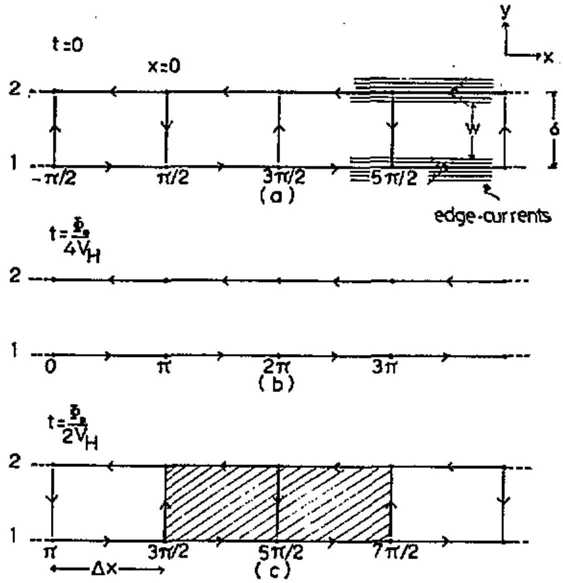

Figure 1. Variation of the phase difference φ_{12} in space and time controls the direction (up or down) of the flow of the phase-driven transverse current between sides 1 and 2 in accordance with the equation $J = J_c \sin \varphi_{12}$. The system current flows in the x-direction in the form of two edge currents. In (a), at time $t = 0$, those values of φ_{12} are shown at a regular interval in x that corresponds to the maximum transverse current, $\pm J_c$; in (b), at $t = \Phi_0/(4V_H)$, the J is zero at all points in the x-direction; and in (c), at $t = \Phi_0/(2V_H)$, the $J = \pm J_c$ at the points shown and the overall situation is the reverse of that in (a). The shaded portion in (c) corresponds to the area $\delta 2\Delta x$ enclosed in a closed loop performed by an electron that starts its trajectory at $x = 0$ on side 2 at $t = 0$. After completing the loop, the electron undergoes a phase change of 2π and thus the loop encloses a flux quantum Φ_0.

In (11), v_x is fixed by the magnitude of the system current that is maintained at an arbitrary value, say around 1 nA, in a typical QHE experiment. The value of v_y, and therefore that of δ, is deduced with the help of (11). In this way the dimensions of the rectangular loop that encloses one Φ_0 are fixed by the externally fixed parameter v_x. With the help of available information we can make an order of magnitude calculation of v_x, and from that deduce the dimensions of the loop approximately and then calculate the period of oscillation ΔB.

The carrier concentration is about $6 \times 10^{15}\ \mathrm{m}^{-2}$ [2]. So, 7.75×10^7 electrons shall lie along a line 1 m wide and normal to the direction of the system current. Some of these will be flowing to the right and the others to the left—their difference $|n_r - n_l|$ will constitute the system current. We do not know $|n_r - n_l|$; all we know is that the system current is about 1 nA in the experiment under consideration [2]. Taking $|n_r - n_l|$ to be of order $10^7\ \mathrm{m}^{-1}$, we calculate v_x as,

$$v_x = 1\ \mathrm{nA}/|n_r - n_l|e \simeq 0.6 \times 10^3\ \mathrm{m\,s^{-1}} \simeq 10^3\ \mathrm{m\,s^{-1}}. \qquad (12a)$$

Then,

$$v_y \simeq 50\ \mathrm{m\,s^{-1}}. \qquad (12b)$$

Thus we find that the electrons carrying the transverse AC move rather sluggishly.

Now we turn to the calculation of ΔB. Suppose flux Φ is penetrating the $\delta x L$ strip in between the two edge currents (L being the length of the system). Then,

$$B(\delta x L) = \Phi \equiv m\Phi_0 \tag{13a}$$

when exactly m loops of the kind discussed above (each enclosing one Φ_0) fit into the $\delta x L$ strip.

Now B is increased by ΔB such that one more flux quantum is added into the $\delta x L$ strip, so

$$(B + \Delta B)(\delta x L) = \Phi + \Phi_0. \tag{13b}$$

Consequently

$$\Delta B = \Phi_0/\delta x L = 4V_{\mathrm{H}}/v_y L \tag{14}$$

the separation between two successive minima of the R_{xx}—it was discussed in VS that whenever the $\delta x L$ strip will enclose an integral number of Φ_0 the R_{xx} will be at its minimum. Equation (14) gives $\Delta B \simeq 0.01$ T for plateau $i = 2$ at $B \simeq 13$ T. Its comparison with the observed value of 0.065 T can be considered good in view of the fact that we do not know the value of $|n_{\mathrm{r}} - n_{\mathrm{l}}|$ in (12) apart from its order of magnitude. The V_{H}-dependence of ΔB is correctly given by (14)—ΔB reduces by a factor of 2 in going from plateau $i = 2$ to $i = 4$ (which corresponds to $V_{\mathrm{H}} \to V_{\mathrm{H}}/2$) for a fixed value of B. The experiment [2] shows the same V_{H} dependence.

The central result of this paper is the relation (11). The important information it gives is that the width of the region between the two edge currents over which the AC flows is decided by the externally controlled parameter v_x. For the given set of parameters it is found that the electrons move back and forth between the two edges with an unexpectedly low speed. This leads to an AC of high frequency ($\sim 10^{10}\,\mathrm{s}^{-1}$) and small wavelength ($\sim 10^{-8}$ m). In the experiment under consideration [2], the B is kept fixed at $\sim$13 T and the Fermi level is moved, from one plateau to another, by changing the carrier concentration while the system current is kept fixed at $\sim$1 nA. Thus to go from the plateau $i = 2$ to $i = 4$ keeping B the same, more electrons are pumped into the system but their speed, v_x, is reduced. Consequently v_y increases (according to (11)) and therefore, the wavelength of the AC and δ increase. This explains physically why the ΔB decreases in going from the $i = 2$ to the $i = 4$ plateau. We can also intuitively understand why the oscillations are not observed for plateaux $i > 4$. As i increases, the δ increases and, therefore, the electrons carrying the AC will have to move from deeper into the bulk of an edge current into the bulk of the other edge current. This increases the probability of scattering of these electrons with those that carry the edge currents; also, there will be more interactions of these electrons with impurities. Consequently the area of the rectangular loop may not remain the same in different regions of the $\delta x L$ strip which will be detrimental to the periodicity of the R_{xx}-oscillations.

Note that the natural requirement for the AC to flow is that δ should be larger than the separation between the edge currents. We can work out the condition for the AC to be set up from (11). It is easily found to be,

$$(4V_{\mathrm{H}}/BI)|n_{\mathrm{r}} - n_{\mathrm{l}}|e \geqslant w \tag{15}$$

where I is the system current and w is the separation between the edge currents. This condition can be useful only if we can know $|n_{\mathrm{r}} - n_{\mathrm{l}}|$ and w in an experiment. Interesting experiments can be designed, and oscillations in R_{xx} of the desired periodicity can be produced if $|n_{\mathrm{r}} - n_{\mathrm{l}}|$ and w can be learned as well as controlled in an experiment.

Thanks are due to Y Imry for discussions and helpful comments.

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